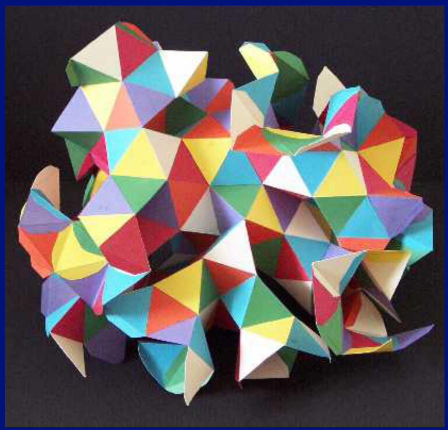
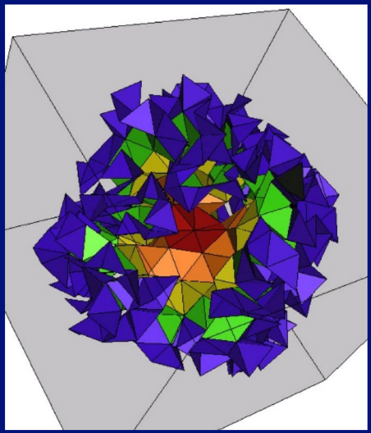
Bridges Baltimore, July 2015

Large, “7-Around” Hyperbolic Disks

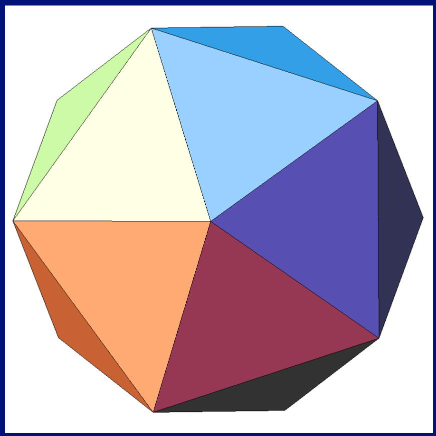
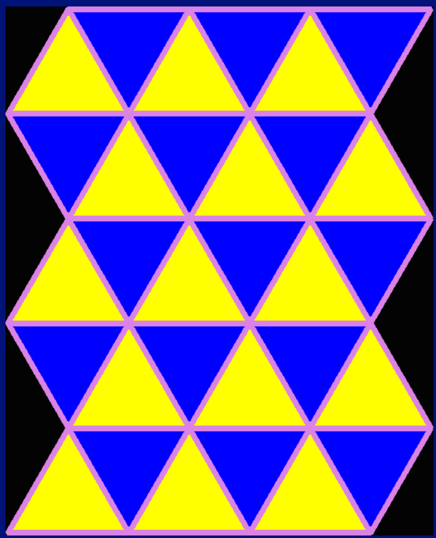
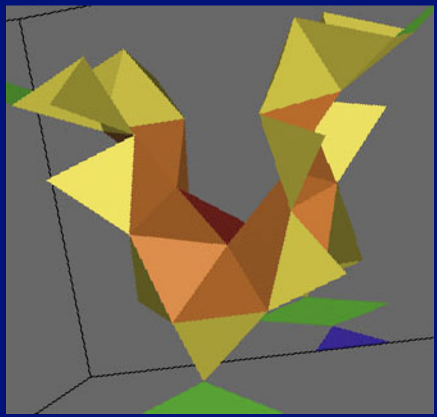




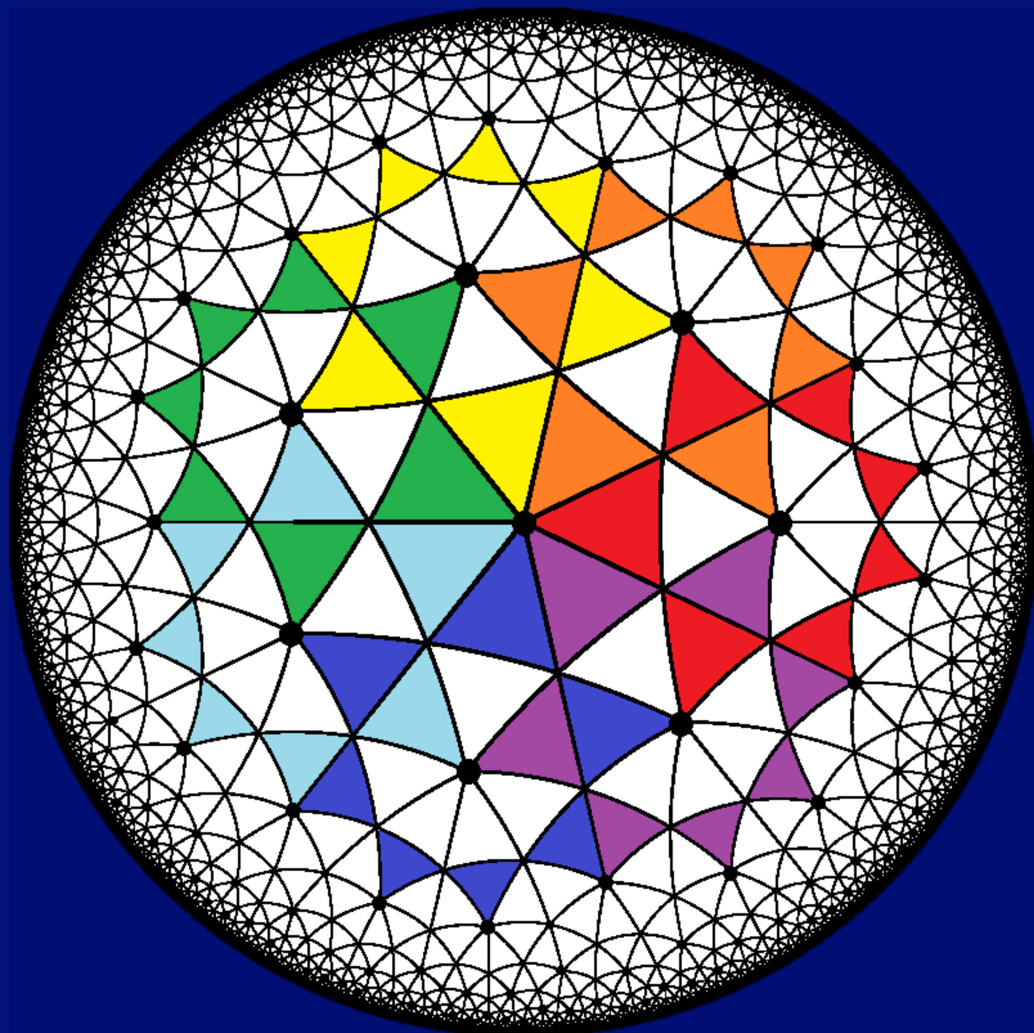
Sean Jeng Liu, Young Kim,
Raymond Shiau, Carlo H. Séquin

University of California, Berkeley

Assembling Equilateral Triangles

5 per vertex:	6 per vertex:	7 per vertex:
		
pos. curved: → Icosahedron	flat: → a plane	neg. curved: → hyperbolic

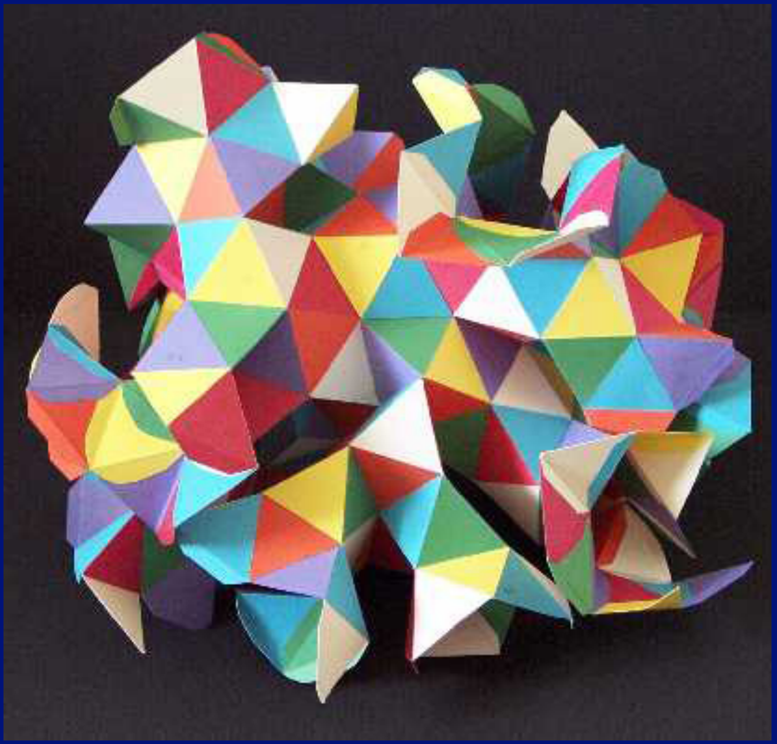
Hyperbolic Surface: Poincaré Disk Model



- Scaling allows to accommodate infinitely many triangles.

How much of that infinite tiling can we accommodate with equilateral triangles ?

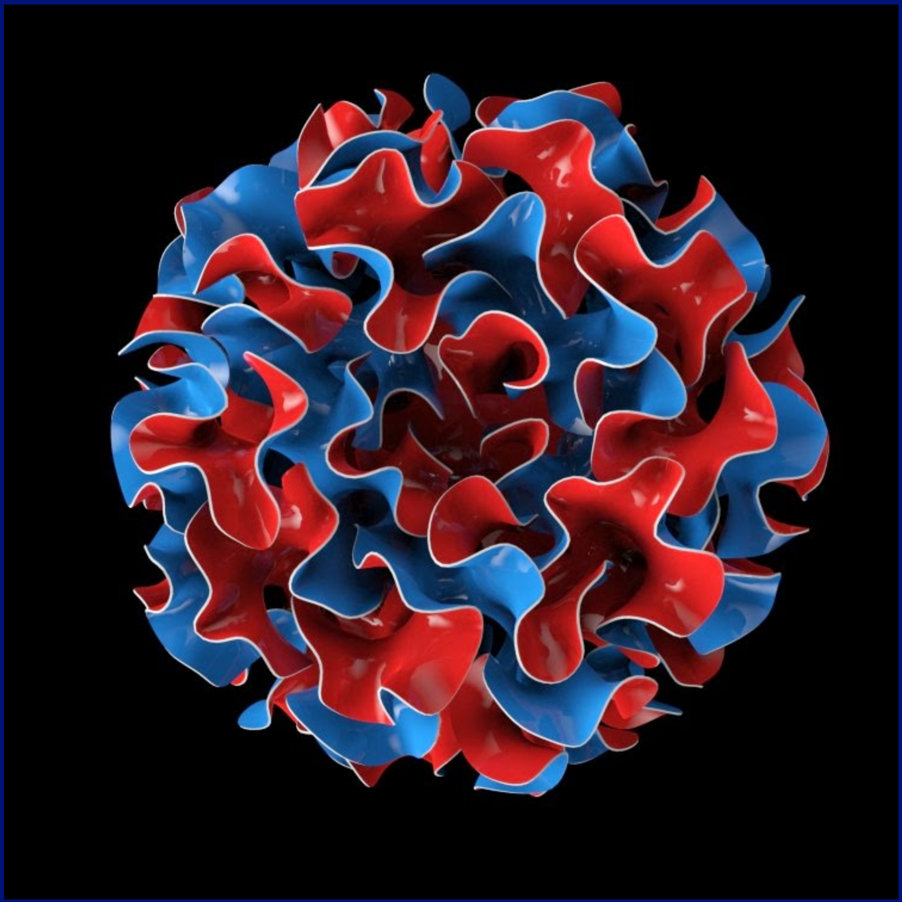
Amy Ione & CW Tyler



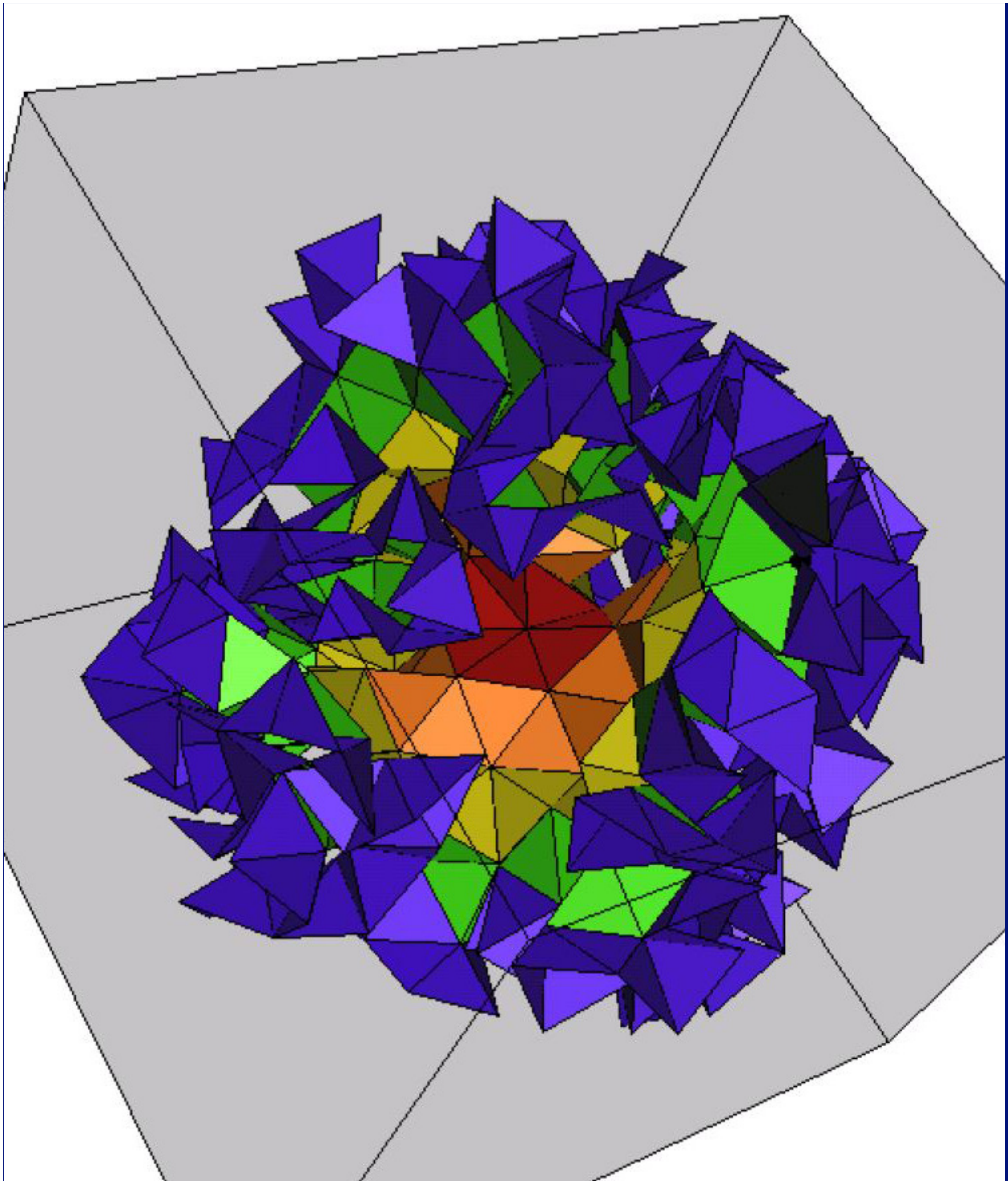
David Richter

Better Luck with Soft Materials

Gabriele Meyer



(posted by Loren Serfass)



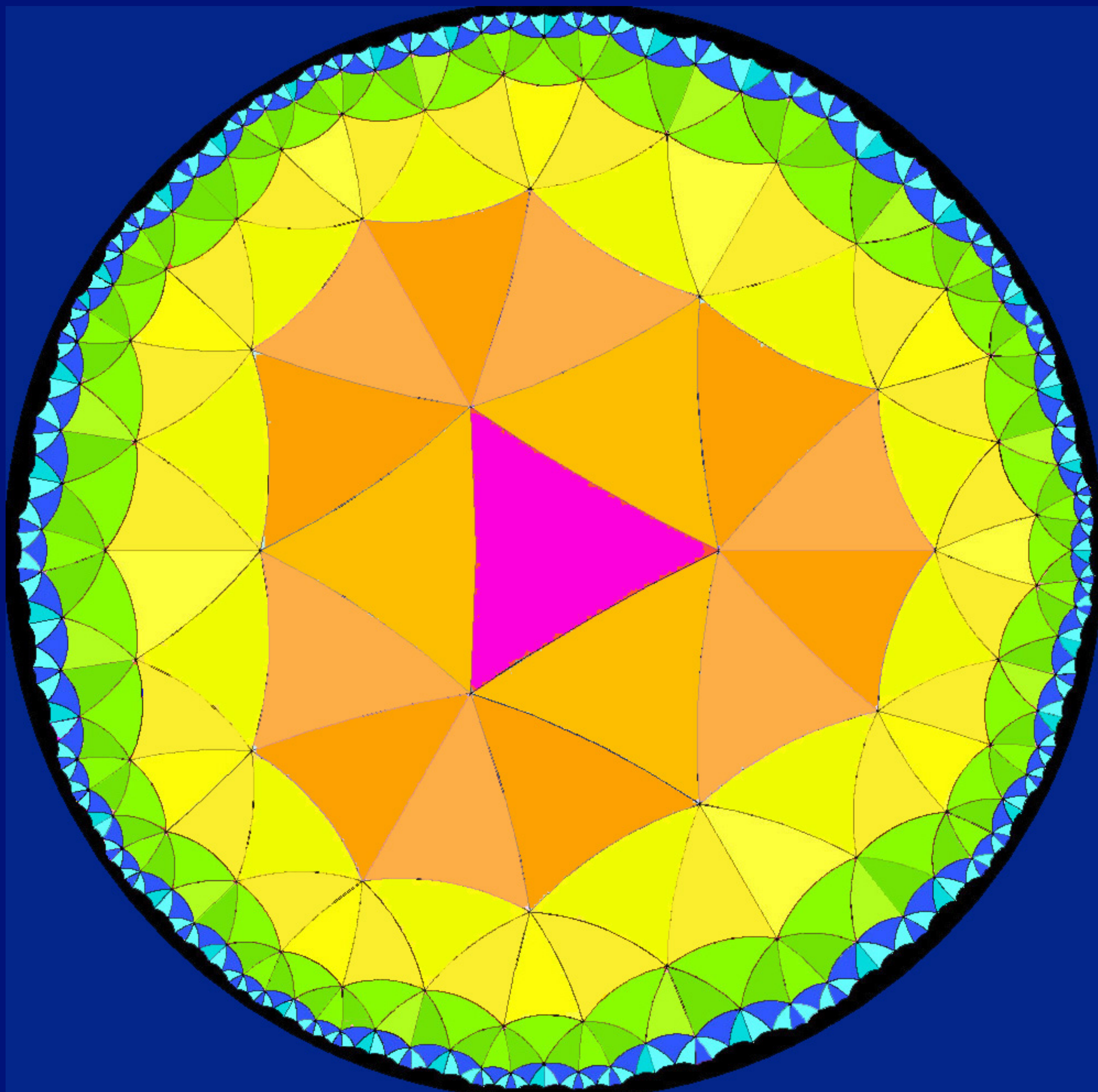
Computer Model by Mark Howison (2007)

Best result:

810 Triangles

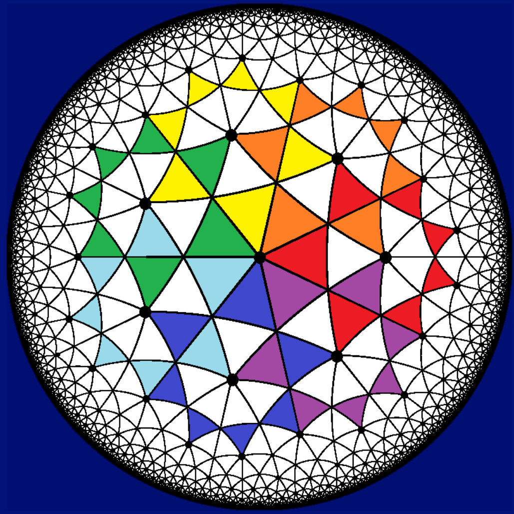
(20 hrs CPU time)

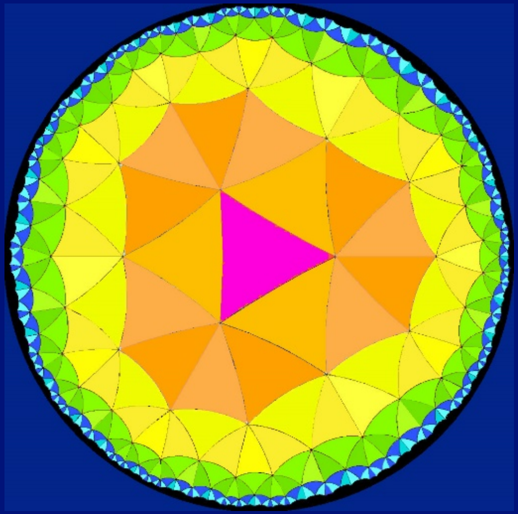
Extending the Disk . . .



- By adding full annuli – one at a time, with ever more triangles …
- Exploit: 6-fold D3-symmetry

New Approach: Add 6-fold Symmetry

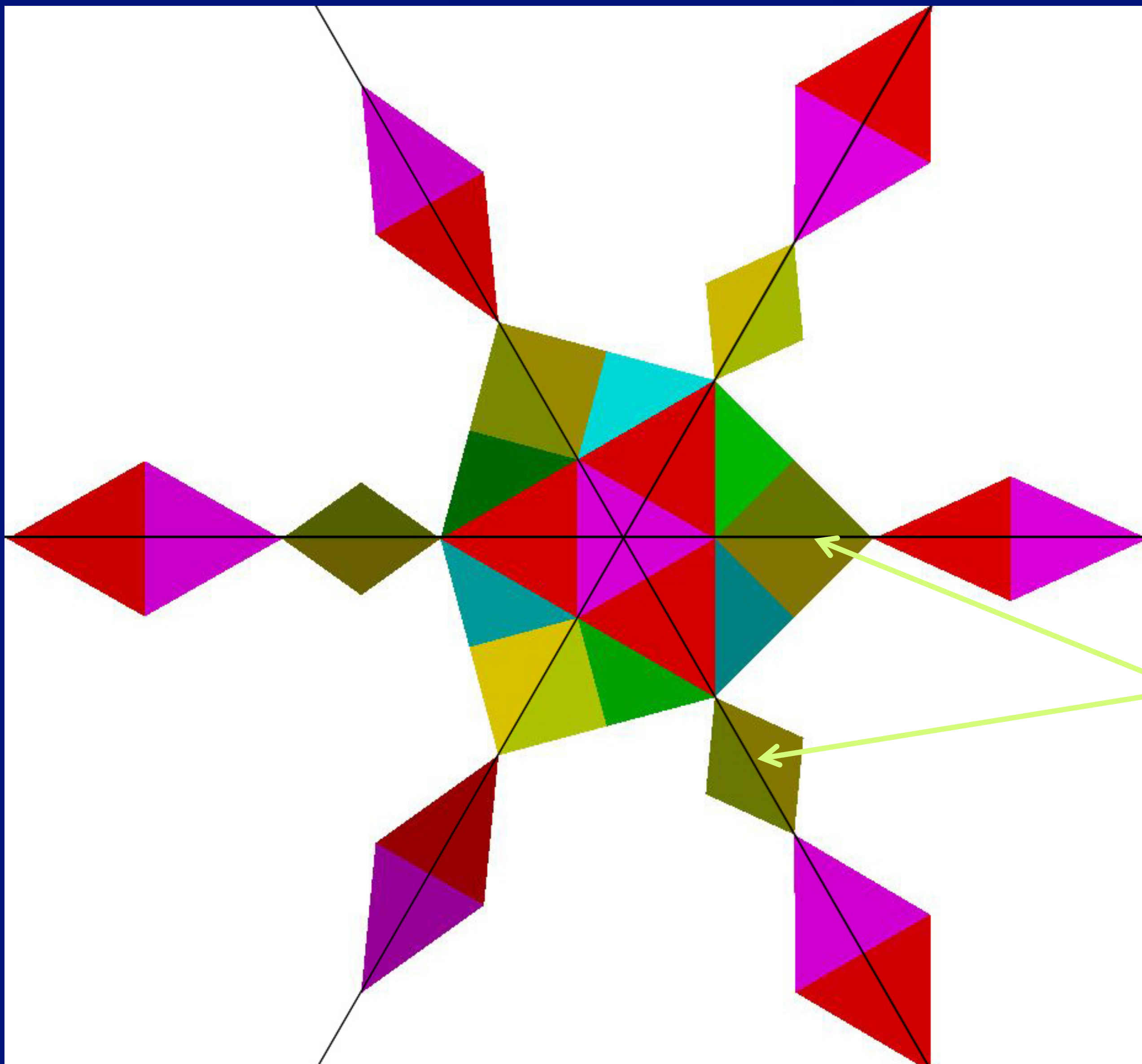




Ring	Faces (old)	Cumul.Faces	Color [1]	Ring	Faces (new)	Cumul.Faces
0	7	7	red	0	1	1
1	28	35	orange	1	15	16
2	77	112	yellow	2 = init.core	45	61
3	219	322	green	3 = swath #1	120	181
4	574	896 →**810**	blue	4 = swath #2	315	496
5	1568	2464	purple	5 = swath #3	825	1321
6	4284	6748	white	6	2160	3481→**2197**

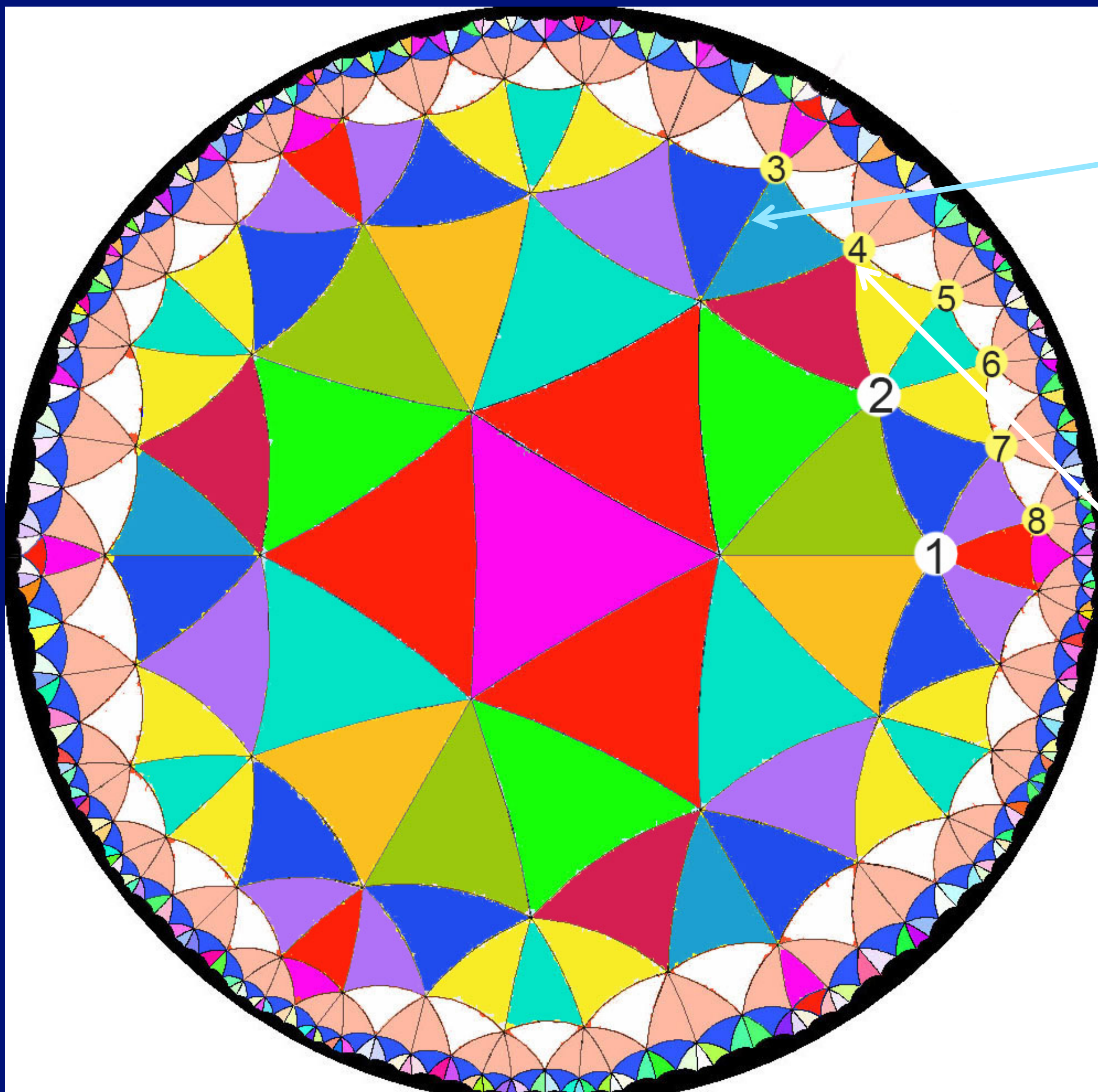
Howison’s annuli: starting with a central vertex. – Our new annuli: starting with a central triangle.

Starting with a Symmetrical Core



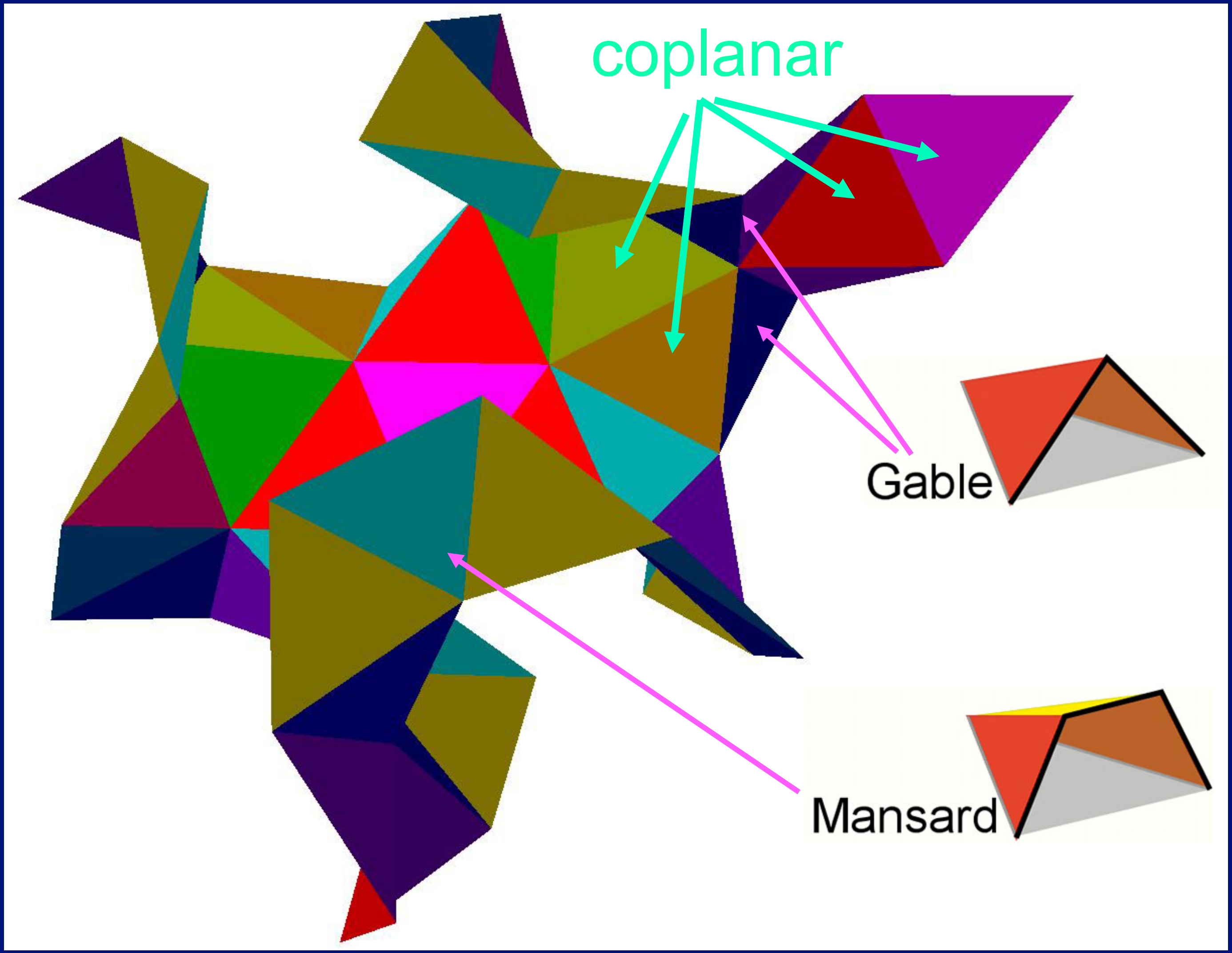
- D3 symmetry forces some constraints:
- The 4 central triangles are coplanar!
- Yellow-olive edges lie on symm. axes; adj. triangles are coplanar.

Constructing an Extended Core



- Blue-teal edges lie on symm. axes. The two triangles are coplanar!
- Give #4 & #2 the same sign for the z-value to make "nice, looping arch"

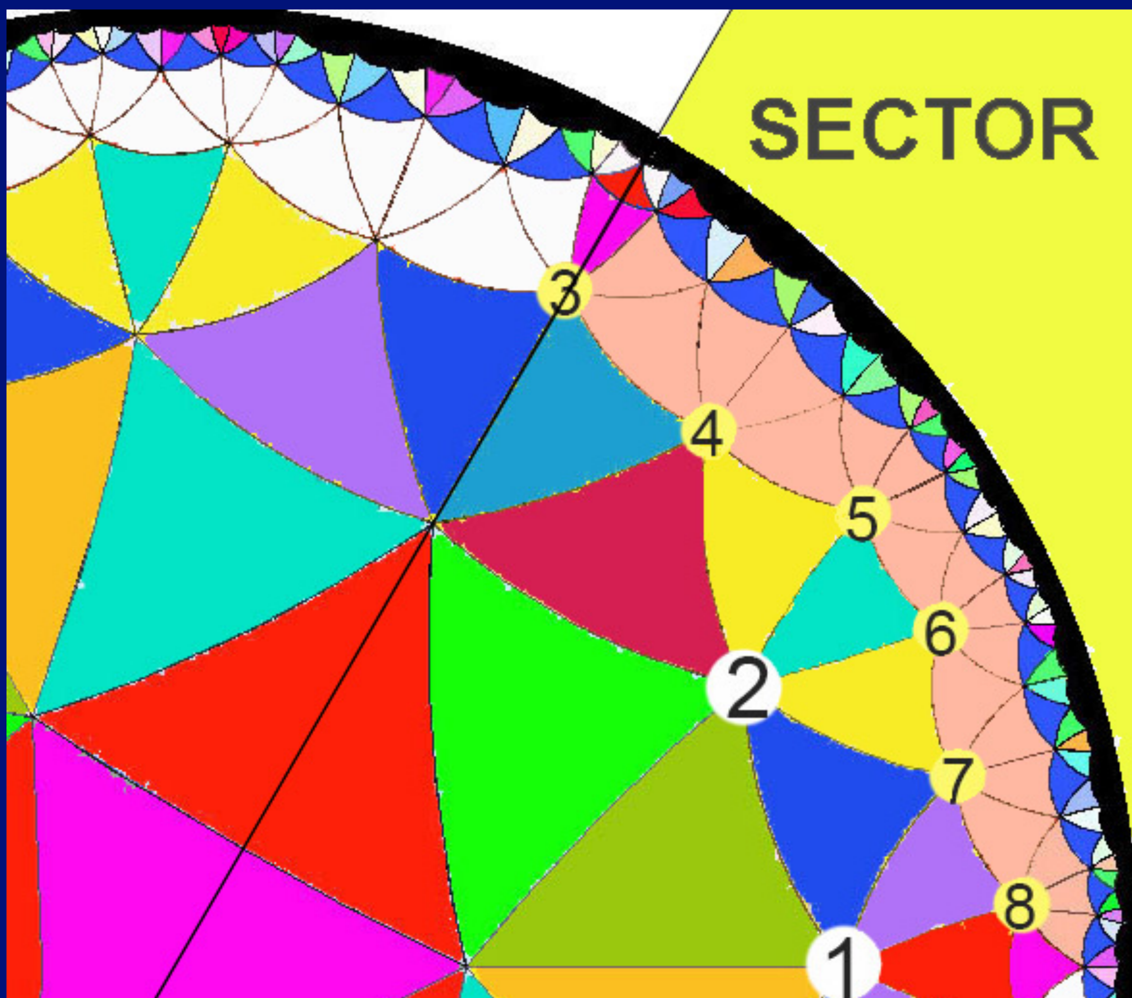
Manually Constructed, Extended Core



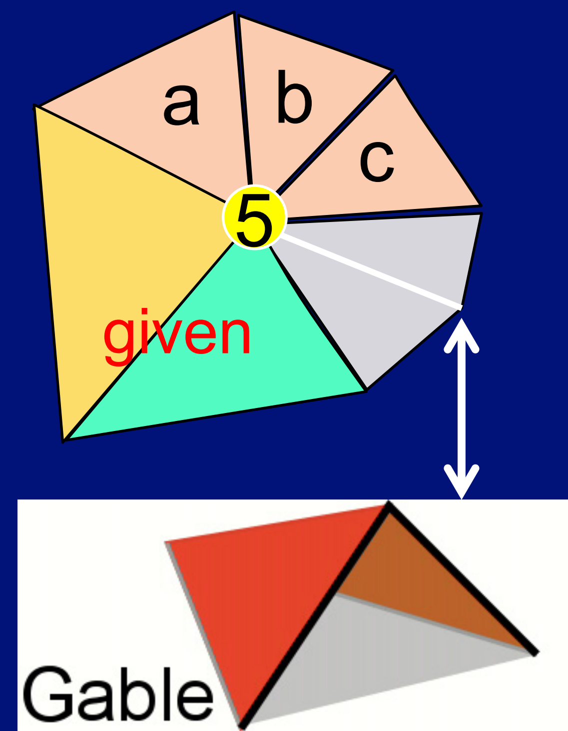
→ 61 triangles with D3 symmetry with nice undulating border.

Step-by-Step Construction

- Complete one vertex at a time: “3”, “4”, “5”, “6”, “7”, “8” in orange “swath #1” throughout a 60° sector.



e.g. vertex “5”:
add “a”, “b”, “c” ind.;
last two as a “gable.”



Interference Checking & Back-Tracking

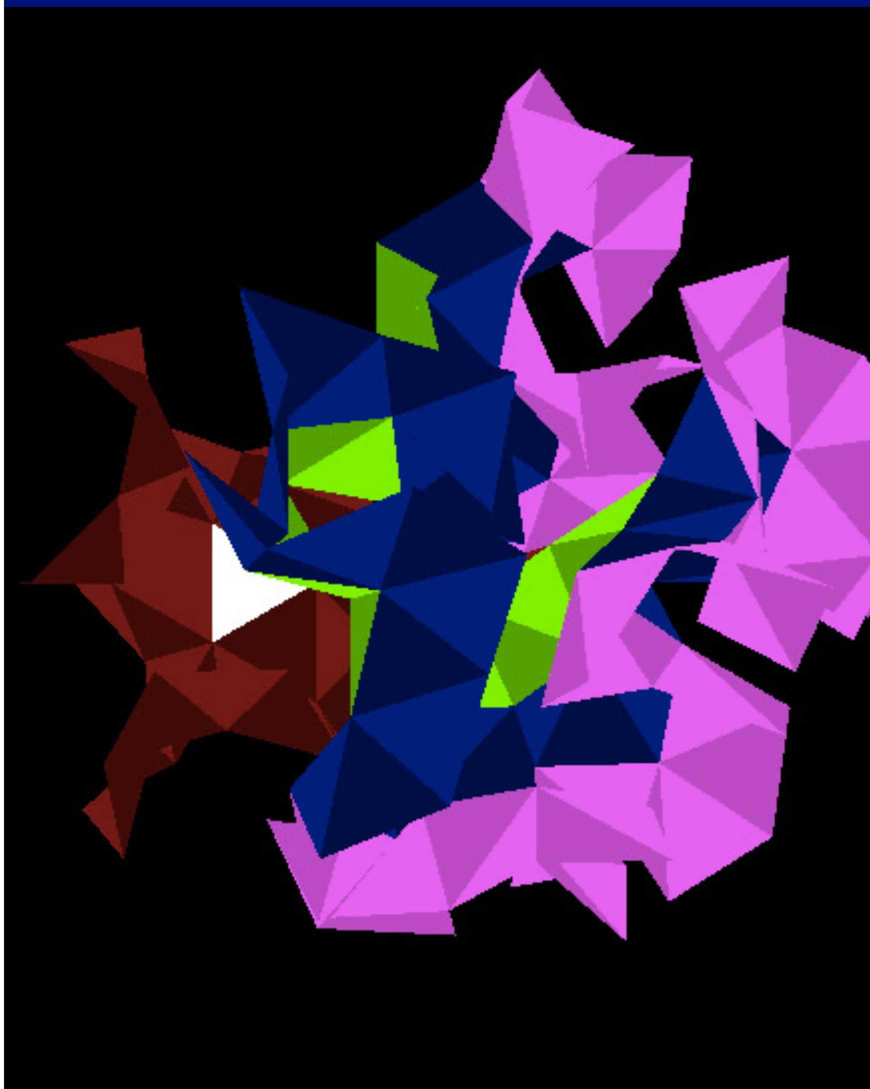
- Interference:
  - Check for intersections between triangles;
  - Apply conservative proximity check for inner swath;
  - Apply the extensive intersection test for outer swaths
- Back-track:
  - If new triangle fails to meet heuristic guidelines or criteria

Overall Strategy: 1 Annulus at a Time

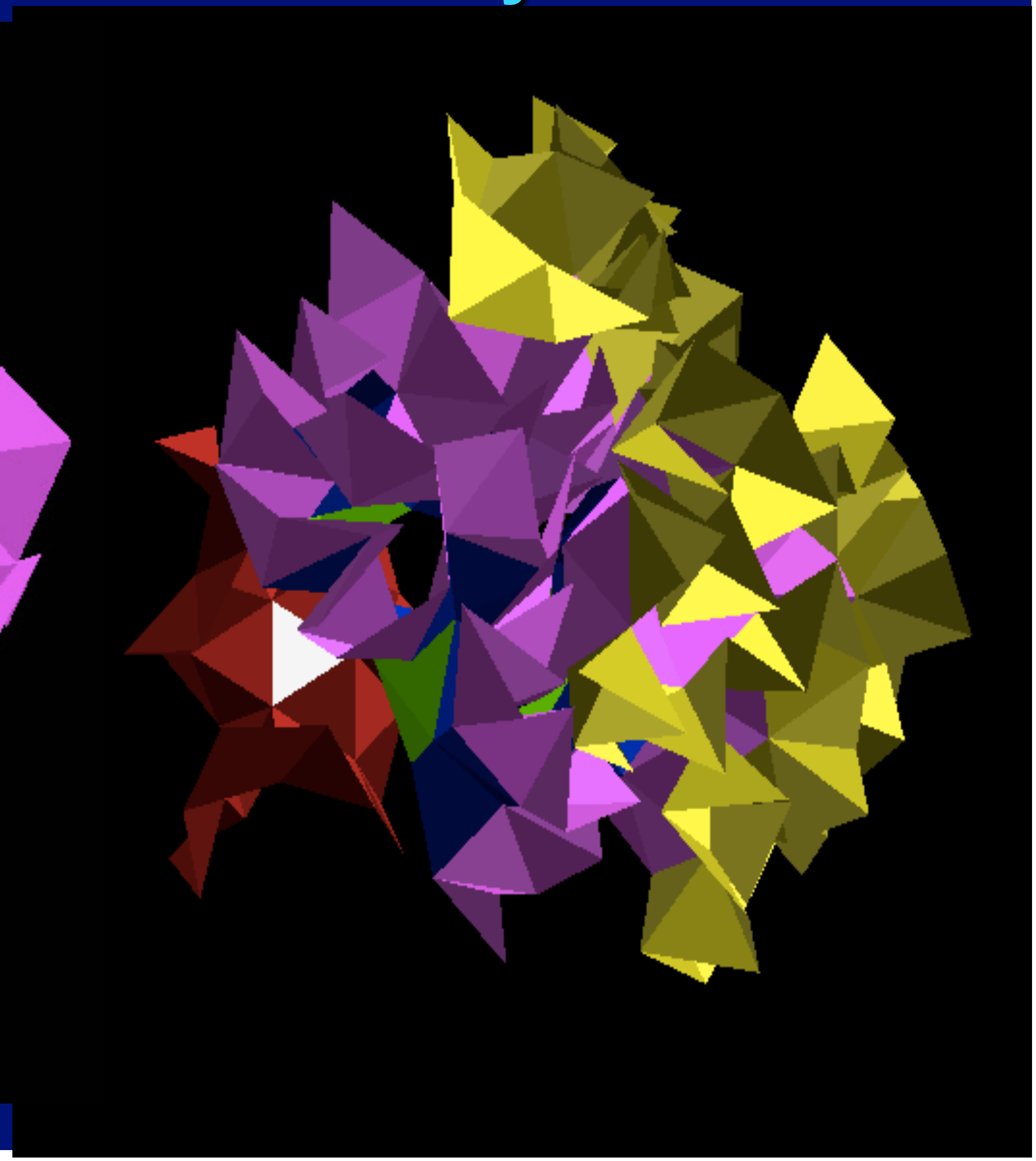
- We only have to construct one 60° sector of the whole disk, which then gets replicated 6 times.
- We construct this one "swath" (= 1/6 annulus) at a time; we try to construct a "good" swath, one that leaves most space for subsequent one.
- Such a good swath gets added to the "core"; it now acts as a starting point for the next swath.

Results

May 2015

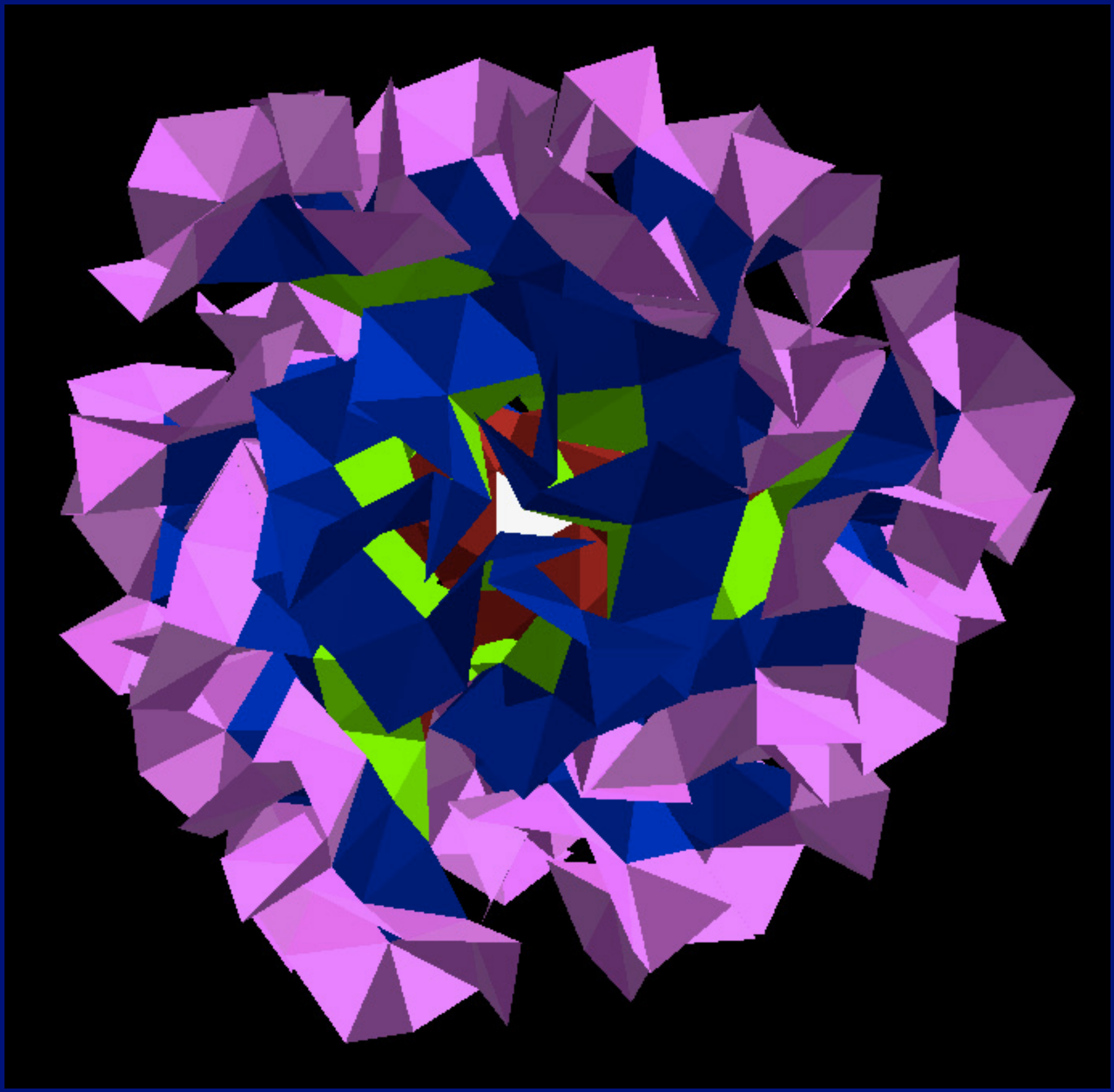


July 2015

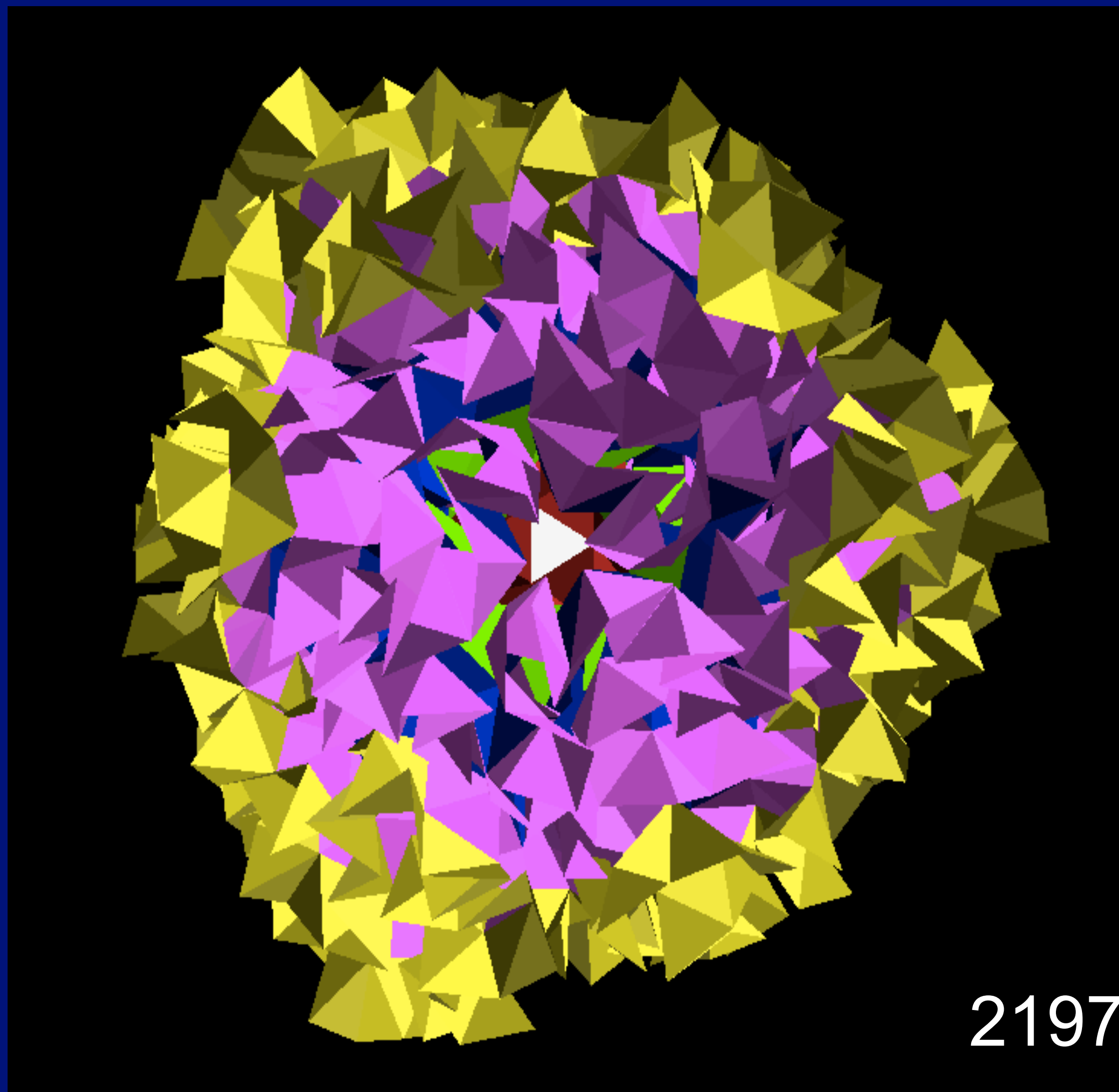


Video

Fully Instantiated Disk (May 2015)



Final Result at Conference Time



2197 triangles

Conclusions

- Computers are useful and powerful;
  but brute-force approaches may only get limited results.
- Use your brain to gain an understanding of the problem,
  and the tailor your search to make use of such insights.
- A good combination of the two approaches
  can then result in a more effective search,
  reducing computation time exponentially.
- This is often true in engineering problems relying on
  simulated annealing or on genetic algorithms.